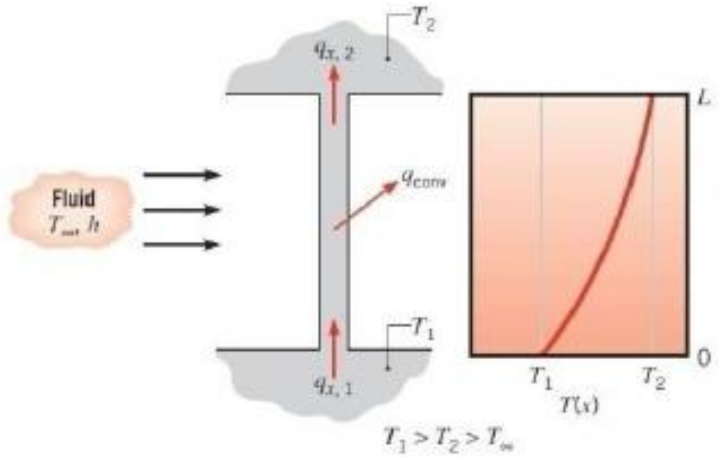
UNIT-II

One Dimensional Transient Conduction Heat Transfer
Theory of Fins

Heat Transfer from Extended Surfaces

Extended surface: solid that experiences energy transfer by conduction within its boundaries, as well as energy transfer by convection and/or radiation between its boundaries and the surround-

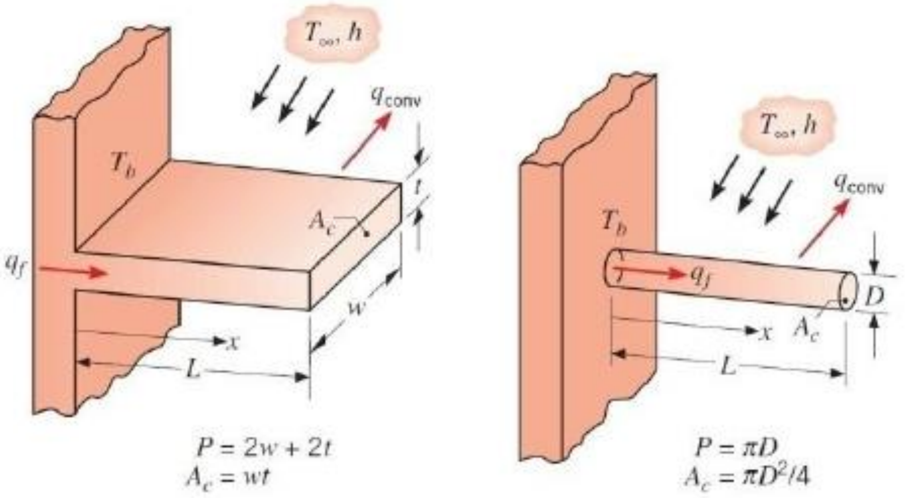


ings.
strut is used to provide mechanical support to two walls at different T. A temperature gradient in the x-direction sustains heat transfer by conduction internally, at the same time there is energy transfer by convection from the surface.

Fin Material and Applications

- k of the fin material has a strong effect on the temperature distribution along the fin and therefore influences the degree to which the heat transfer rate is enhanced.
- Ideally, the fin material should have a large k to minimize temperature variations from its base to its tip.
- In the limit of infinite thermal conductivity, the entire fin would be at the temperature of the base surface, thereby providing the maximum possible heat transfer enhancement.
- The arrangement for cooling engine heads on motorcycles and lawn-mowers
- For cooling electric power transformers
- The tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner

Fins of Uniform Cross Sectional Area



- $T(0) = T_b$
- A_c is constant, $dA_c/dx = 0$
- $A_s = Px$ where x is measured from base, P is fin perimeter
- $dA_s/dx = P$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

Excess temperature, θ

$$\theta(x) = T(x) - T_\infty$$

$$d\theta/dx = dT/dx$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where $m^2 = \frac{hP}{kA_c}$

The above equation is a linear, homogeneous, second-order differential equation with constant coefficients. The general solution is of the form:

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

It is necessary to specify appropriate BCs for C_1 and C_2.

One such condition may be specified in terms of the temperature at the base of the fin $(x = 0)$:

$$\theta(0) = T_b - T_\infty = \theta_b$$

The second condition, specified at the fin tip $(x = L)$, may correspond to any one of the four different physical conditions:

A. h at the fin tip
B. Adiabatic condition at the fin tip
C. Prescribed temperature maintained at the fin tip
D. Infinite fin (very long fin)

A. Infinite fin (very long fin): As $L \to \infty$, $\theta_L \to 0$

B. Adiabatic condition at the fin tip

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

C. h at the fin tip

$$hA_c[T(L) - T_\infty] = -kAc\left.\frac{dT}{dx}\right|_{x=L} \implies h\theta(L) = -k\left.\frac{d\theta}{dx}\right|_{x=L}$$

D. Prescribed temperature maintained at the fin tip: $\theta(L) = \theta_L$

Uniform Cross- Sectional Fin : Summary

Temperature distribution & heat loss for fins of uniform cross-section

Tip Cond.	at $x = L$	$\frac{\theta}{\theta_b}$	q_f
Infinite fin	$\theta(L) = 0$	e^{-mx}	M
Adiabatic	$\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh[m(L-x)]}{\cosh mL}$	$M \tanh mL$
Convection	$h\theta_L = -k\left.\frac{d\theta}{dx}\right\|_{x=L_c}$	$\frac{\cosh[m(L_c-x)]}{\cosh mL_c}$	$M \tanh mL$

$$m = \sqrt{\frac{hP}{kA_c}}; \qquad M = \sqrt{hPkA_c}\,\theta_b; \qquad L_c = L + \frac{A_c}{P}$$

Fin Efficiency

- The temperature of the fin will be T_b at the fin base and gradually decrease towards the fin tip.
- Convection from the fin surface causes the temperature at any cross-section to drop somewhat from the midsection toward the outer surfaces.
- However, the cross-sectional area of the fins is usually very small, and thus the temperature at any cross-section can be considered to be uniform.
- Also, the fin tip can be assumed for convenience and simplicity to be adiabatic by using the corrected length for the fin instead of the actual length.

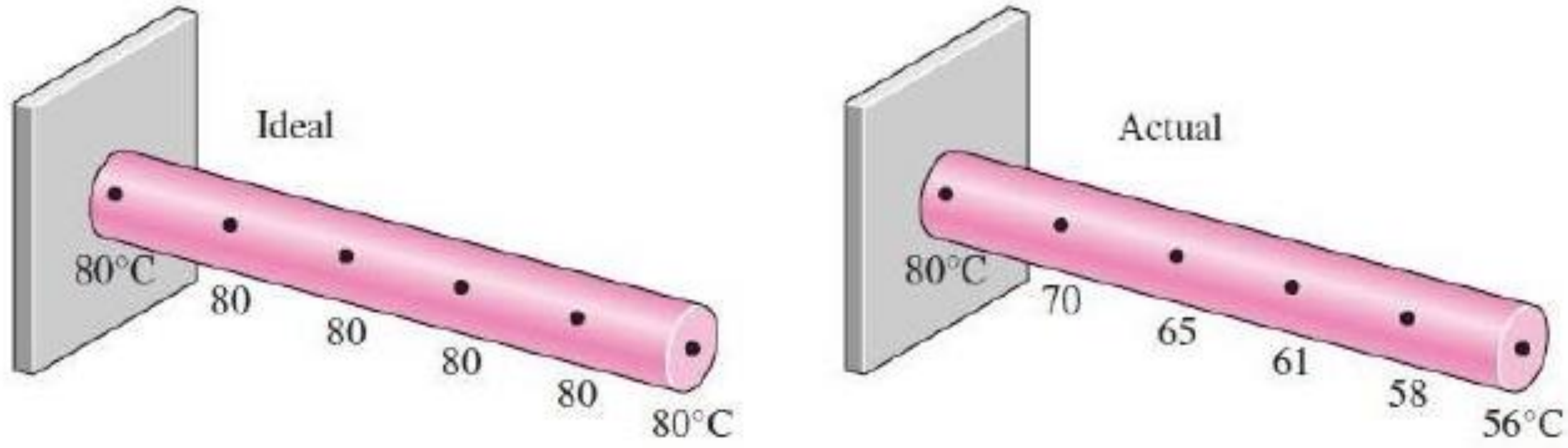
In the limiting case of zero thermal resistance or infinite k, the temperature of fin will be uniform at the value of T_b. The heat transfer from the fin will be maximum in this case ($k \rightarrow \infty$):

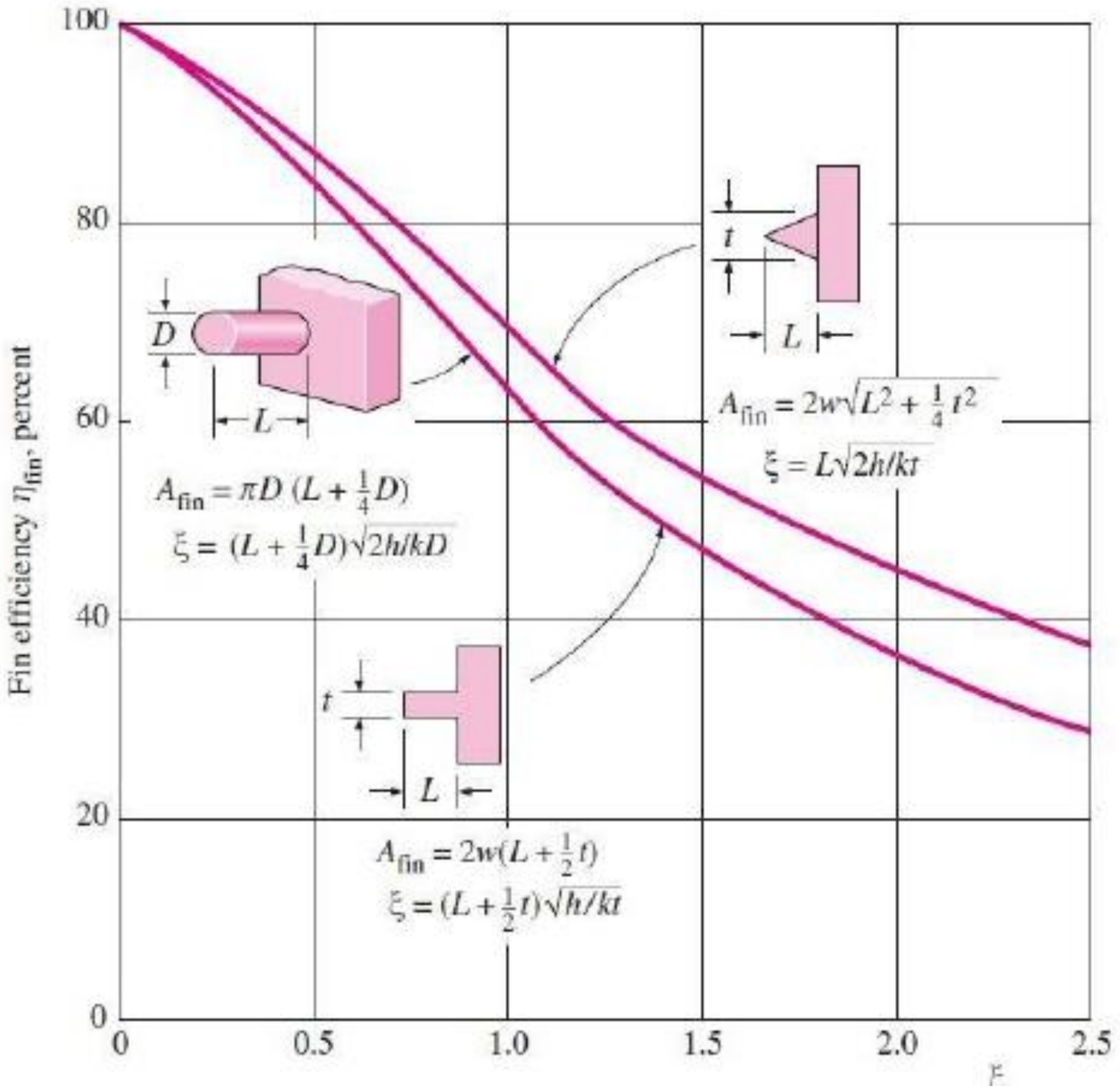
$$q_{fin,max} = hA_{fin}(T_b - T_\infty)$$

In reality, however, the temperature of the fin will drop along the fin and thus the heat transfer from the fin will be less because of the decreasing $[T(x) - T_\infty]$ toward the fin tip.

To account for the effect of this decrease in temperature on heat transfer, we define fin efficiency as:

$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$


Ideal
80°C
80
80
80
80
80°C
Actual
80°C
70
65
61
58
56°C


100
80
60
40
20
0
0
0.5
1.0
1.5
2.0
2.5
ξ
Fin efficiency η_{fin}, percent
D
L
$A_{fin} = \pi D (L + \frac{1}{4}D)$
$\xi = (L + \frac{1}{4}D)\sqrt{2h/kD}$
t
L
$A_{fin} = 2w\sqrt{L^2 + \frac{1}{4}t^2}$
$\xi = L\sqrt{2h/kt}$
t
L
$A_{fin} = 2w(L + \frac{1}{2}t)$
$\xi = (L + \frac{1}{2}t)\sqrt{h/kt}$

Overall Effectiveness

We can also define an overall effectiveness for a finned surface as the ratio of the total q from the finned surface to the q from the same surface if there were no fins:

$$\varepsilon_{fin,overall} = \frac{q_{fin}}{q_{nofin}} = \frac{h(A_{unfin} + \eta_{fin}A_{fin})(T_b - T_\infty)}{hA_{nofin}(T_b - T_\infty)}$$

A_{nofin} is the area of the surface when there are no fins
A_{fin} is the total surface area of all the fins on the surface
A_{unfin} is the area of the unfinned portion of the surface.

$\varepsilon_{fin,overall}$ depends on number of fins per unit length as well as ε_{fin} of individual fins.
$\varepsilon_{fin,overall}$ is a better measure of the performance than ε_{fin} of individual fins.